Using DeMoivre's Theorem

Find $(-3+3i\sqrt{3})^5$ using DeMoivre's Theorem.

We can relate a complex number to its trigonometric form:

$$z = a + bi$$

$$z = r(\cos\theta + i\sin\theta)$$

We also have formulas for the modulus r and the argument theta:

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a}$$

First find the modulus and the argument. *(Note: Using a calculator on inverse tangent might not give us the smallest positive angle. For example, one calculator might give us -60 degrees for this problem. So we want to double-check with the unit circle. We should use positive 120 degrees.)*

$$r = \sqrt{(-3)^2 + (3\sqrt{3})^2}$$
$$r = \sqrt{9+27}$$
$$r = \sqrt{36}$$
$$r = 6$$

$$\tan\theta = \frac{3\sqrt{3}}{-3}$$
$$\tan\theta = -\frac{\sqrt{3}}{1}$$
$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) = 120^\circ$$

Precalculus B Lesson 6.6
De Moivre’s Theorem

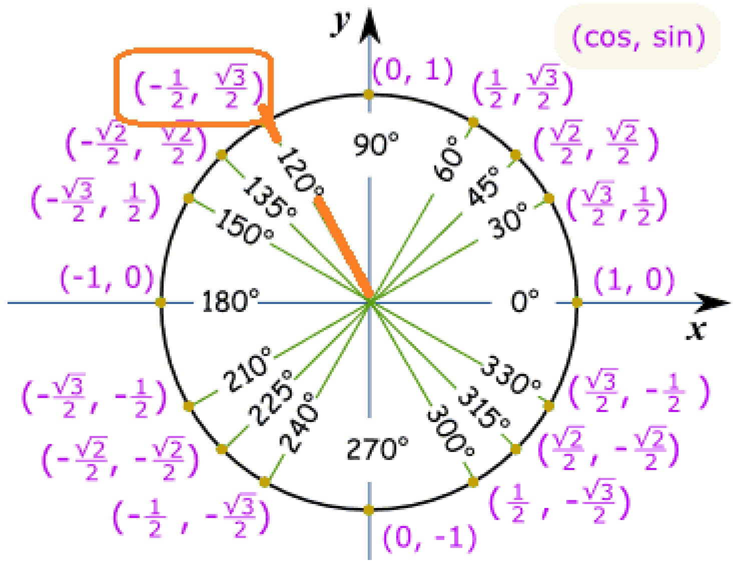
Review for using the unit circle:

$$\tan = \frac{opp}{adj}$$

$$\tan 120^{\circ} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$\tan 120^{\circ} = \frac{\sqrt{3}}{2} \bullet \left(-\frac{2}{1}\right)$$

$$\tan 120^{\circ} = -\frac{\sqrt{3}}{1}$$



We can set the complex form equal to the trigonometric form:

$$(-3+3i\sqrt{3}) = 6(\cos 120^{\circ} + i \sin 120^{\circ})$$

Also raise both sides to the fifth exponent:

$$(-3+3i\sqrt{3})^5 = \left[6(\cos 120^{\circ} + i \sin 120^{\circ})\right]^5$$

$$(-3+3i\sqrt{3})^5 = 6^5(\cos 120^{\circ} + i \sin 120^{\circ})^5$$

Then use De Moivre’s Theorem:

$$(-3+3i\sqrt{3})^5 = 6^5\left(\cos(5\cdot 120^{\circ}) + i \sin(5\cdot 120^{\circ})\right)$$

Distribute and then use a calculator for each term. Round to the nearest tenth:

$$(-3+3i\sqrt{3})^5 = 6^5\cos(5\cdot 120^{\circ}) + 6^5 i \sin(5\cdot 120^{\circ})$$

$$(-3+3i\sqrt{3})^5 = 7776\cdot\cos(5\cdot 120^{\circ}) + 7776\cdot i \sin(5\cdot 120^{\circ})$$

$$(-3+3i\sqrt{3})^5 = -3888 - 6743.2i$$